Markscheme

May 2016

Further mathematics

Higher level

Paper 1

17 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM™ Assessor instructions and the document "**Mathematics HL: Guidance for e-marking May 2016**". It is essential that you read this document before you start marking. In particular, please note the following:

- Marks must be recorded using the annotation stamps. Please check that you are entering marks for the right question.
- If a part is **completely correct**, (and gains all the "must be seen" marks), use the ticks with numbers to stamp full marks.
- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, it **must** be recorded using **all** the annotations.
- All the marks will be added and recorded by RM™ Assessor.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc.*, do **not** split the marks.
- Once a correct answer to a question or part-question is seen, ignore further working. However, if further working indicates a lack of mathematical understanding do not award the final ***A1***. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal. However, if the incorrect decimal is carried through to a subsequent part, and correct ***FT*** working shown, award ***FT*** marks as appropriate but do not award the final ***A1*** in that part.

Examples

	Correct answer seen	**Further working seen**	**Action**
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	Award the final ***A1*** *(ignore the further working)*
2.	$\frac{1}{4}\sin 4x$	$\sin x$	Do not award the final ***A1***
3.	$\log a - \log b$	$\log(a-b)$	Do not award the final ***A1***

3 *N* marks

*Award **N** marks for **correct** answers where there is **no** working.*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

4 Implied marks

*Implied marks appear in **brackets eg (M1)**, and can only be awarded if **correct** work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

5 Follow through marks

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s). To award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part.*

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).
- Within a question part, once an error is made, no further **dependent *A*** marks can be awarded, but ***M*** marks may be awarded if appropriate.
- Exceptions to this rule will be explicitly noted on the markscheme.

6 Misread

*If a candidate incorrectly copies information from the question, this is a misread (**MR**).*
*A candidate should be penalized only once for a particular misread. Use the **MR** stamp to indicate that this has been a misread. Then deduct the first of the marks to be awarded, even if this is an **M** mark, but award all others so that the candidate only loses one mark.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

Example: for differentiating $f(x) = 2\sin(5x-3)$, the markscheme gives:

$f'(x) = \left(2\cos(5x-3)\right)5 \quad \left(=10\cos(5x-3)\right)$ ***A1***

Award ***A1*** for $\left(2\cos(5x-3)\right)5$, even if $10\cos(5x-3)$ is not seen.

10 Accuracy of Answers

*Candidates should **NO LONGER** be penalized for an accuracy error **(AP)**.*

*If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures. Please check work carefully for **FT**.*

11 Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

12 Calculators

A GDC is required, but calculators with symbolic manipulation features (for example, TI-89) are not allowed.

Calculator notation
The Mathematics HL guide says:
Students must always use correct mathematical notation, not calculator notation.
Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

13 More than one solution

Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise.

1. (a) (i) 17 is an element not a subset of P ***R1***

(ii) 57 is not a prime number ***R1***

(iii) any demonstration that this is the true statement ***A1***
because every set contains the empty set as a subset ***R1***

[4 marks]

(b) (i) $f(1) = \phi$ ***A1***
because 1 has no prime factors ***R1***

(ii) $f(2310) = f(2 \times 3 \times 5 \times 7 \times 11) (= \{2, 3, 5, 7, 11\})$ ***A1***
$n\left(f(2310)\right) = 5$ ***A1***

[4 marks]

(c) (i) not injective ***A1***
because, for example, $f(2) = f(4) = \{2\}$ ***R1***

(ii) not surjective ***A1***
$f^{-1}(2,3,5,7,11,13)$ does not belong to S because
$2 \times 3 \times 5 \times 7 \times 11 \times 13 > 2500$ ***R1***

Note: Accept any appropriate example.

[4 marks]

Total [12 marks]

2. (a) $\mathrm{P}(B > 15.5) (= \mathrm{P}(Z > 0.5))$ ***(M1)***
$= (1 - 0.69146) = 0.309$ ***A1***

[2 marks]

(b) consider $V = B_1 + B_2 + B_3 + B_4 + B_5 + B_6 + B_7$ ***(M1)***
$\mathrm{E}(V) = 98$ ***(A1)***
$\mathrm{Var}(V) = 63$ or equivalent ***(A1)***

Note: No need to state V is normal.

$\mathrm{P}(V < 100) = \left(\mathrm{P}(Z < \frac{2}{\sqrt{63}} = 0.251976\ldots\right) = 0.599$ ***A1***

[4 marks]

continued…

Question 2 continued

(c) consider $W = R_1 + R_2 + R_3 + R_4 + R_5 - (B_1 + B_2 + B_3 + B_4 + B_5 + B_6 + B_7)$ ***(M1)***

$\mathrm{E}(W) = 2$ ***(A1)***

$\mathrm{Var}(W) = 80 + 63 = 143$ ***(A1)***

$\mathrm{P}(W > 0) = \left(\mathrm{P}\left(Z < \frac{2}{\sqrt{143}}\right)\right)$ ***(M1)***

$= 0.566$ ***A1***

[5 marks]

Total [11 marks]

3. (a) one solution is $x = -2,\ y = -3$ (or $(3,4)$) ***(A1)***

the general solution is

$x = -2 + 5N,\ y = -3 + 7N$ (or $x = 3 + 5M,\ y = 4 + 7M$) ***M1A1***

[3 marks]

(b) a listing of small values of the product ***(M1)***

$\Rightarrow x = -2, y = -3$ (the least positive value of xy is 6) ***A1***

[2 marks]

(c) use of “table” or otherwise to solve

$35N^2 - 29N + 6 = 2014$ (or $35M^2 + 41M + 12 = 2014$) ***(M1)***

obtain $N = 8$ (or $M = 7$) ***(A1)***

$x = 38,\ y = 53$ ***A1***

[3 marks]

Total [8 marks]

4. (a) $\mathrm{H}_0: \mu = 12.4;\ \mathrm{H}_1: \mu > 12.4$ ***A1***

[1 mark]

(b) (i) *t* test is appropriate because the variance (standard deviation) is unknown ***R1***

$\nu = 9$ ***A1***

(ii) $t \geq 1.83$ (5%); $t \geq 1.38$ (10%) ***A1A1***

Note: Accept strict inequalities.

[4 marks]

continued…

Question 4 continued

(c) (i) unbiased estimate of μ is 13.18 ***A1***

Note: Accept 13.2.

unbiased estimate of σ^2 is 2.34 (1.531^2) ***A1***

(ii) $$t_{\text{calc}} = \left(\frac{13.18 - 12.4}{\frac{1.531}{\sqrt{10}}}\right) = 1.61 \text{ or } 1.65$$ ***A1***

[3 marks]

(d) as $1.38 < 1.61 < 1.83$ ***R1***

evidence to accept H_0 at the 5% level, but not at the 10% level ***A1***

Note: Accept the use of the p-value $= 0.0708$.

[2 marks]

Total [10 marks]

5. attempt to find the equation of the tangent at P ***M1***

$y - x_1^3 = 3x_1^2(x - x_1)$ ***A1***

the tangent meets C when

$x^3 - x_1^3 = 3x_1^2(x - x_1)$ ***M1***

attempt to solve the cubic ***M1***

the x-coordinate of Q satisfies

$x^2 + x x_1 - 2x_1^2 = 0$ ***A1***

hence $x_2 = -2x_1$ ***A1***

hence $x_3 = 4x_1$ ***A1***

hence x_1, x_2, x_3 form the first three terms of a geometric sequence with common ratio –2 so the sequence is divergent ***R1AG***

Note: Final ***R1*** is not dependent on final 3 ***A1***s providing they form a geometric sequence.

Total [8 marks]

6. (a) $H_2 = 2H_1 + 1$ ***(M1)***

$= 3;\ H_3 = 7;\ H_4 = 15$ ***A1***

[2 marks]

(b) $H_n = 2^n - 1$ ***A1***

[1 mark]

continued…

Question 6 continued

(c) let $\mathrm{P}(n)$ be the proposition that $H_n = 2^n - 1$ for $n \in \mathbb{Z}^+$

from (a) $H_1 = 1 = 2^1 - 1$ ***A1***

so $\mathrm{P}(1)$ is true

assume $\mathrm{P}(k)$ is true for some $k \Rightarrow H_k = 2^k - 1$ ***M1***

then $H_{k+1} = 2H_k + 1$ ***(M1)***

$= 2 \times (2^k - 1) + 1$ ***A1***

$= 2^{k+1} - 1$

$\mathrm{P}(1)$ is true and $\mathrm{P}(k)$ is true $\Rightarrow \mathrm{P}(k+1)$ is true, hence $\mathrm{P}(n)$ is true for all $n \in \mathbb{Z}^+$ by the principle of mathematical induction ***R1***

Note: Only award the ***R1*** if all earlier marks have been awarded.

[5 marks]

Total [8 marks]

7. (a) consider $\lim\limits_{h \to 0+} f(0+h) = \lim\limits_{h \to 0+} \left(2h^2 - 3h + 1\right)$ ***M1***

$= 1 = f(0)$ ***A1***

$\lim\limits_{h \to 0-} f(0+h) = \lim\limits_{h \to 0-} \left(-3h + 1\right)$ ***M1***

$= 1 = f(0)$ ***A1***

hence f is continuous at $x = 0$ ***AG***

[4 marks]

Note: The $= f(0)$ needs only to be seen once.

(b) consider

$$\lim_{h \to 0+} \left(\frac{f(0+h) - f(0)}{h}\right) = \lim_{h \to 0+} \left(\frac{2h^2 - 3h + 1 - 1}{h}\right)$$ ***M1A1***

$$= \lim_{h \to 0+} \left(\frac{2h^2 - 3h}{h}\right) = -3$$ ***A1***

$$\lim_{h \to 0-} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0-} \frac{-3h + 1 - 1}{h} = -3$$ ***M1A1***

hence f is differentiable at $x = 0$ ***AG***

[5 marks]

Total [9 marks]

8. (a) (i) let (x, y) be a point on C

then $(x+3)^2 + y^2 = k^2\left((x-5)^2 + y^2\right)$ ***M1A1A1***

Note: Award ***M1*** for form of an Apollonius circle, ***A1*** for each side.

rearrange, for example,

$(k^2-1)x^2 - (10k^2+6)x + (k^2-1)y^2 + 25k^2 - 9 = 0$ ***A1***

equate the x-coordinate of the centre as given by this equation to 13:

$\dfrac{5k^2+3}{k^2-1} = 13$ ***M1A1***

obtain $k^2 = 2 \Rightarrow k = \sqrt{2}$ ***A1***

(ii) **METHOD 1**

with this value of k, the equation can be reduced to the form

$(x-13)^2 + y^2 = 128$ ***M1A1***

obtain the radius $= \sqrt{128}\ \left(= 8\sqrt{2}\right)$ ***A1***

METHOD 2

assuming N is the x-intercept of C between A and B

$\dfrac{\mathrm{AN}}{\mathrm{BN}} = \dfrac{16-r}{r-8} = \sqrt{2}$ ***M1A1***

$\Rightarrow r = 8\sqrt{2}$ ***A1***

Note: Accept answers given in terms of k, if no value of k found in (a)(i).

(iii) x-intercepts are $13 \pm 8\sqrt{2}$ ***A1***

[11 marks]

(b) because N lies on the circle it satisfies the Apollonius property

hence $\mathrm{AN} = \sqrt{2}\,\mathrm{NB}$ ***R1***

but as $\mathrm{AM} = \sqrt{2}\,\mathrm{MB}$ ***R1***

by the converse to the angle-bisector theorem ***R1***

$\mathrm{A\hat{M}N} = \mathrm{N\hat{M}B}$ ***AG***

[3 marks]

Total [14 marks]

9. (a) $5982 = 162 \times 36 + 150$ ***M1A1***

$162 = 150 \times 1 + 12$ ***A1***

$150 = 12 \times 12 + 6$

$12 = 6 \times 2 + 0 \Rightarrow \text{gcd is } 6$ ***A1***

[4 marks]

continued…

Question 9 continued

(b) (i) for example, $\gcd(4, 4) = 4$ ***A1***

$4 \neq 2$ ***R1***

so R is not reflexive ***AG***

for example

$\gcd(4, 2) = 2$ and $\gcd(2, 8) = 2$ ***M1A1***

but $\gcd(4, 8) = 4 \ (\neq 2)$ ***R1***

so R is not transitive ***AG***

(ii) **EITHER**

even numbers ***A1***

not divisible by 6 ***A1***

OR

$\{2 + 6n : n \in \mathbb{N}\} \cup \{4 + 6n : n \in \mathbb{N}\}$ ***A1A1***

OR

$2, 4, 8, 10, \ldots$ ***A2***

[7 marks]

Total [11 marks]

10. (a) **METHOD 1**

$2^n = (3-1)^n$ ***M1***

$= 3^n + n3^{n-1}(-1) + \frac{n(n-1)}{2}3^{n-2}(-1)^2 + \ldots + (-1)^n$ ***A1***

since all terms apart from the last one are divisible by 3 ***R1***

$2^n \equiv (-1)^n \pmod 3$ ***AG***

METHOD 2

attempt to reduce the powers of $2 \pmod 3$ ***M1***

$2^0 = 1 \pmod 3;\ 2^1 = -1 \pmod 3;\ 2^2 = 1 \pmod 3;\ 2^3 = -1 \pmod 3 \ldots$ ***A1***

since $1 \pmod 3 \times 2 = -1 \pmod 3$ and $-1 \pmod 3 \times 2 = 1 \pmod 3$ the result can be generalized ***R1***

$2^n \equiv (-1)^n \pmod 3$ ***AG***

[3 marks]

continued…

Question 10 continued

(b) the binary number $N = (a_n a_{n-1} \ldots a_2 a_1 a_0)_2$ has numerical value

$a_0 \times 1 + a_1 \times 2 + a_2 \times 2^2 + \ldots + a_n \times 2^n$ ***A1***

$N = \left(a_0 - a_1 + a_2 - \ldots (-1)^n a_n\right) (\bmod 3)$ ***M1A1***

hence divisibility of N by 3 coincides with statement of question ***AG***

[3 marks]

(c) $\mathrm{ABBA}_{16} = 10 \times 16^3 + 11 \times 16^2 + 11 \times 16 + 10 \times 1$ ***(A1)***

$N = (1010)_2 \times 2^{12} + (1011)_2 \times 2^8 + (1011)_2 \times 2^4 + (1010)_2 \times 2^0$ ***(M1)(A1)***

Note: Award ***M1*** for expressing A and B in binary.

$N = (1010101110111010)_2$ ***A1***

[4 marks]

Total [10 marks]

11. suppose R is the midpoint of BC ***M1***

Note: The first mark is for initiating a relevant discussion for "if" or "only if" by Ceva's theorem.

$\frac{\mathrm{AP}}{\mathrm{PB}} \times \frac{\mathrm{BR}}{\mathrm{RC}} \times \frac{\mathrm{CQ}}{\mathrm{QA}} = 1$ ***A1***

$\Rightarrow \frac{\mathrm{AP}}{\mathrm{PB}} = \frac{\mathrm{AQ}}{\mathrm{QC}}$ or equivalent ***A1***

$\Rightarrow \frac{\mathrm{PB}}{\mathrm{AP}} + 1 = \frac{\mathrm{QC}}{\mathrm{AQ}} + 1$ ***(M1)***

$\Rightarrow \frac{\mathrm{AP} + \mathrm{PB}}{\mathrm{AP}} = \frac{\mathrm{AQ} + \mathrm{QC}}{\mathrm{AQ}}$

$\Rightarrow \frac{\mathrm{AB}}{\mathrm{AP}} = \frac{\mathrm{AC}}{\mathrm{AQ}}$ ***A1***

$\Rightarrow$ triangles APQ and ABC are similar with common base angles ***R1***

so PQ is parallel to BC ***AG***

statement of the converse ***A1***

the argument is reversible ***R1AG***

Total [8 marks]

12. (a) Accept any valid reasoning:

Example 1:
$(1, 0, 0)$ lies on the plane, however linear combinations of this do not (for example $(2, 0, 0)$) ***R1***
hence the position vectors of the points on the plane do not form a vector space ***AG***

Example 2:
the given plane does not pass through the origin (or the zero vector is not the position vector of any point on the plane) ***R1***
hence the position vectors of the points on the plane do not form a vector space ***AG***

[1 mark]

(b) (i) (the set of position vectors is non-empty)

let $\boldsymbol{x}_1 = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}$ be the position vector of a point on the plane and $a \in \mathbb{R}$

then the coordinates of the position vector of $a\boldsymbol{x}$ satisfy the equation for the plane because $ax_1 - ay_1 - az_1 = a(x_1 - y_1 - z_1) = 0$ ***M1A1***

let $\boldsymbol{x}_2 = \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix}$ be the position vector of another point on the plane

consider $\boldsymbol{x}_3 = \boldsymbol{x}_1 + \boldsymbol{x}_2$

then the coordinates of $\boldsymbol{x}_3 = \begin{pmatrix} x_3 = x_1 + x_2 \\ y_3 = y_1 + y_2 \\ z_3 = z_1 + z_2 \end{pmatrix}$ satisfy ***M1***

$x_3 - y_3 - z_3 = (x_1 + x_2) - (y_1 + y_2) - (z_1 + z_2)$
$= 0$ ***A1***
subspace conditions established ***AG***

Note: The above conditions may be combined in one calculation.

continued…

Question 12 continued

(ii) if $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ is the position vector of a second point on the plane orthogonal to the given vector, then ***(M1)***

$a - b - c = 0$ and $a + 2b - c = 0$ ***(A1)(A1)***

for example $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ completes the basis ***A1***

(iii) the basis for $(\mathbb{R}^2)$ can be augmented to an orthogonal basis for $\mathbb{R}^3$ by adjoining $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \times \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ ***(M1)***

$= \begin{pmatrix} 2 \\ -2 \\ -2 \end{pmatrix}$ ***A1***

(iv) attempt to solve $\alpha \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \gamma \begin{pmatrix} 2 \\ -2 \\ -2 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ -2 \end{pmatrix}$ ***M1***

obtain $\alpha = \beta = \gamma = 1$ ***A2***

[13 marks]

Total [14 marks]

13. (a) using $\left(\dfrac{t^n - 1}{t - 1}\right) = 1 + t + t^2 + \dots t^{n-1}$ ***M1***

$G_n(t) = 0 + \dfrac{t}{n} + \dfrac{t^2}{n} + \dfrac{t^3}{n} + \dots \dfrac{t^n}{n} + 0 \times t^{n+1} + 0 \times \dots$ ***A1A1***

Note: ***A1*** for the non-zero terms, ***A1*** for the observation that all other terms are zero.

the statement that the coefficient of t^k gives $\mathrm{P}(X_n = k)$ ***R1***

hence $\mathrm{P}(X_n = k) = \begin{cases} \dfrac{1}{n} & \text{for } 1 \le k \le n \\ 0 & \text{otherwise} \end{cases}$ ***AG***

[4 marks]

continued…

Question 13 continued

(b) $\mathrm{E}(X_n) = 0 \times 0 + 1 \times \frac{1}{n} + 2 \times \frac{1}{n} + 3 \times \frac{1}{n} + \ldots n \times \frac{1}{n} + (n+1) \times 0 + \ldots \times 0$

(M1)(A1)

$= \frac{1}{n} \times \sum_{k=1}^{k=n} k$

$= \frac{1}{n} \times \frac{1}{2} n(n+1) = \frac{n+1}{2}$ ***A1***

Note: Accept use of $G'(1)$.

[3 marks]

(c) X_{n-1} and X_{n+1} are independent $\Rightarrow \mathrm{E}(X_{n-1} \times X_{n+1}) = \mathrm{E}(X_{n-1}) \times \mathrm{E}(X_{n+1})$ ***M1***

$= \frac{n}{2} \times \frac{n+2}{2}$ ***A1***

required to solve $n^2 < 6n$ (or $n + 2 < 8$)) ***M1***

solution: $(2 \le)\ n < 6$ ***A1***

[4 marks]

Total [11 marks]

14. (a) (i) $\boldsymbol{M}^2 = \boldsymbol{MM}$ only exists if the number of columns of $\boldsymbol{M}$ equals the number of rows of $\boldsymbol{M}$ ***R1***

hence $\boldsymbol{M}$ is square ***AG***

(ii) apply the determinant function to both sides ***M1***

$\det(\boldsymbol{M}^2) = \det(\boldsymbol{M})$

use the multiplicative property of the determinant

$\det(\boldsymbol{M}^2) = \det(\boldsymbol{M})\det(\boldsymbol{M}) = \det(\boldsymbol{M})$ ***(M1)***

hence $\det(\boldsymbol{M}) = 0$ or 1 ***A1***

[4 marks]

(b) (i) attempt to calculate $\boldsymbol{N}^2$ ***M1***

obtain $\begin{pmatrix} -a^2 & 2a^2 \\ -a^2 & 2a^2 \end{pmatrix}$ ***A1***

equating to $\boldsymbol{N}$ ***M1***

to obtain $a = -1$ ***A1***

continued…

Question 14 continued

(ii) $\boldsymbol{N} = \begin{pmatrix} -1 & 2 \\ -1 & 2 \end{pmatrix}$

$\boldsymbol{N} - \lambda\boldsymbol{I} = \begin{pmatrix} -1-\lambda & 2 \\ -1 & 2-\lambda \end{pmatrix}$ ***M1***

$(-1-\lambda)(2-\lambda) + 2 = 0$ ***(A1)***

$\lambda^2 - \lambda = 0$ ***(A1)***

λ is 1 or 0 ***A1***

(iii) let $\lambda = 1$

to obtain $\begin{pmatrix} -1 & 2 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$ or $\begin{pmatrix} -2 & 2 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ ***M1***

hence eigenvector is $\begin{pmatrix} x \\ x \end{pmatrix}$ ***A1***

let $\lambda = 0$

to obtain $\begin{pmatrix} -1 & 2 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ ***M1***

hence eigenvector is $\begin{pmatrix} 2y \\ y \end{pmatrix}$ ***A1***

Note: Accept specific eigenvectors.

[12 marks]

Total [16 marks]